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DYNAMICS OF UNSTABLE OSCILLATIONS IN A SUPERSONIC JET

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Until quite recently the major tools in the important and practically significant branch of gas dynamics related to pulsation processes in supersonic jets were experimental methods. Other methods which would permit a deeper understanding of the essence of such phenomena still remain in a developmental stage. This assertion is also fully applicable to mathematical modeling methods, a fact related not so much to reticence on the part of scientists or insufficiently developed mathematical techniques, as to fundamental difficulties in modeling a wide class of transient gas dynamic phenomena. It is the essence of mathematical methods that they are designed to circumvent unavoidable difficulties in precise calculation while still retaining the fundamental features of the true solution. In this light the major difficulty often arises in constructing a correct physical model which accurately describes a phenomenon or class of phenomena, since within a complex combination of interrelated processes, selection of some single realization sufficiently simple for mathematical realization is often fraught with the danger of destroying cause–effect relationships of the true process.

However the level of development of experimental methods does not fully correspond to the level of problems posed regarding the nature of pulsations and their relationship to noise generation processes, since the imperfections of measurement techniques in high-gradient and often dirty flows under experimental conditions limit us to measurements of averaged or integral values, allowing definition of only one or the other property of a phenomenon, knowledge of the basic features of which remains over the horizon. In this situation a consolidation of various methods each providing an incomplete representation of the whole may serve as a link to describing the complete mechanism of the process or phenomenon.

The goal of the present study is to study possibilities of realizing the mechanisms of three-wave resonant interactions involving filling of the spectrum of perturbations which define the so-called wide-band noise of the jet.

We will consider the possibility of describing the experimentally observed longitudinal evolution of coarse-scale shear disturbances in a supersonic jet within the framework of a weakly nonlinear stability theory. The actual situation to be considered is the following: in the initial portion of a supersonic flow two well-defined peaks exist in the perturbation amplitude–frequency spectrum [1, 2] with a narrow range of frequencies (Strouhal numbers $\mathrm{Sh} = 2\pi\omega\bar{r}_0/a(U_0 = 0)$). The frequency ratio of these peaks is close to 1:2. There can be no doubt that oscillations at the lower frequency, the intensity of which varies little down the flow, have an effect on oscillations at the doubled frequency, encouraging their rapid growth, so that the latter soon become the dominant mode, defining the carrier frequency of the jet's acoustic radiation. It is known reliably that for Mach numbers greater than or equal to 1.5 the latter are spiral modes with an azimuthal wave number $n = 1$ or -1, although identification of disturbances indicates that near the nozzle section both spiral and axisymmetric $n = 0$ modes exist. Calculations of linear characteristics [3] show that both types of oscillations are unstable and have comparable increments.

Thus, if we propose that this pattern of longitudinal perturbation dynamics is defined by the interaction of oscillations at different frequencies, the nonlinear mechanism by which azimuthal large scale waves at high frequencies are produced may provide the key to an understanding of the physical processes of a free flow at supersonic speed.

In the present study we will consider the mechanism of resonant wave splitting. It is known, for example, that in a free (compressed, or uncompressed) infrasonic shear layer a mechanism of subharmonic resonant interaction is realized for two-dimensional infrasonic perturbations, for which the phase velocities of harmonics and subharmonics coincide [4, 5]. A preliminary study of linear characteristics of unstable oscillations for a supersonic jet has shown that at least for the Mach number studied ($M_0 = 1.5$) the condition of two-wave subharmonic resonance is not realized in the initial portion of the jet. Consequently, the resonant mechanism, if exists, must be of a more complicated nature, for example, a three-wave one.

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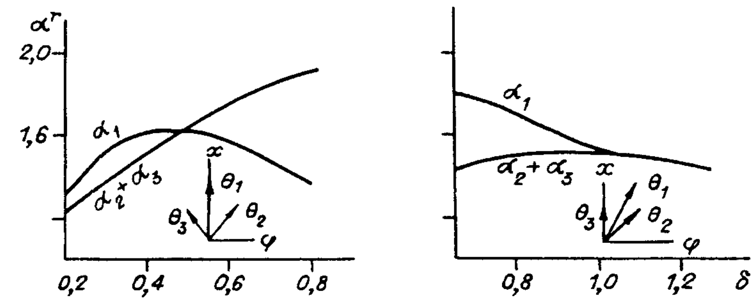

Fig. 1

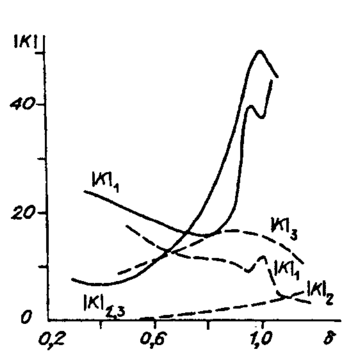

Fig. 2

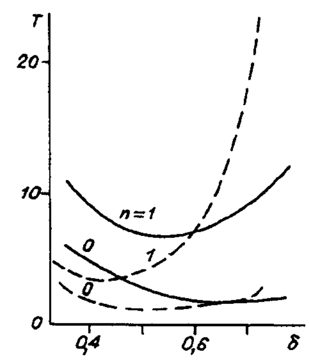

Fig. 3

The present study will consider two triads for which phase synchronism conditions are satisfied. For the wave system in cylindrical coordinates r, φ, x

$$p' = \sum_{j=1}^{3} B_j(\varepsilon x, \varepsilon t) p_j(r) e^{i\theta_j}, \ \theta = \alpha x - \omega t + n\varphi + \psi, \ \alpha = \alpha^r + i\alpha^i \tag{1}$$

the resonant triplets relating different-mode disturbances at the higher harmonic frequency ω_1 (Sh_1) and the lower subharmonic frequency ω_2 (Sh_2) can be of two types: a pair of synchronized spiral waves of the Mach type (superpositions of modes n_2 = 1 and $n_3 = -1$) at the subharmonic frequency and an axisymmetric mode $n_1 = 0$ at the harmonic frequency. We will arbitrarily call such a triplet symmetric in contrast to the asymmetric one formed by different-mode oscillations $n_2 = 1$ and $n_3 = 0$ at the lower frequency ω_2 and a spiral wave $n_1 = 1$ at the double frequency ω_1.

The degree of synchronization of the linear waves in such systems is shown in Fig. 1. In the symmetric triad the resonance is close to a single-point type ($\delta = 0.47$, δ being the thickness of the mixing layer). Nonlinear coupling, and hence wave interaction, can appear in the direct vicinity of the nozzle section, with this coupling decreasing further down the flow, and for $\Delta\alpha^r \sim 0.2$ wave development occurs autonomously. In the asymmetric triad, on the other hand, the nonlinear coupling is established in the middle of the initial segment and maintained at significantly larger longitudinal distances. Lower linear increments α^i accompany this coupling. We will note that for $M_0 = 1.5$, $x = 0.228/\delta$ [6].

Mathematical modeling of interaction in resonant triads by numerical integration of the amplitude equations for $A_j = B_j e^{-\alpha^i_j x}$ was considered in detail in [6, 7]. The final form of those expression is

$$\frac{\partial A_j}{\partial x} = -\alpha^i_j A_j + \varepsilon h K_j A_l A_m, \ j, l, m = 1, 2, 3, \ j \neq m \neq l. \tag{2}$$

where the first term on the right describes linear amplitude growth of the corresponding wave, and a correction is produced by the nonlinear coefficients K_j, formed of quadratic terms of the form $u' \, \delta u'/\delta x$, ..., appearing in the Euler system and the conservation equation for the compressed gas on the basis of which the modeling is carried out; h is the resonant coupling coefficient (for exact resonance h = 1, while with increase in detuning $\Delta\alpha^r$, $h \to 0$).

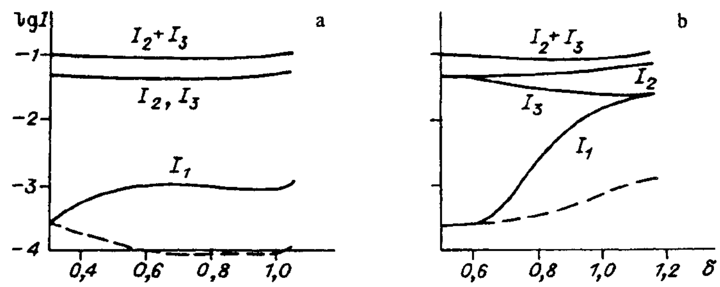

Fig. 4

Figure 2 shows absolute values $|K_j|$ for the triads considered (solid lines, symmetric; dashes, asymmetric triplets). On the whole these are complex functions of variable sign. The character of their distribution down the flow permits the *a priori* conclusion that for the first triad growth K in the advection direction is accompanied simultaneously by decrease in h, and in regions where for the second triad h ~ 1 no significant growth in amplitude will occur due to reduction in K, so that on the whole the degree of nonlinear interaction will be limited and of the same order in both cases.

It develops that the interaction has the character of energy redistribution between the interacting waves. It was found that amplitude increase at the higher frequency always occurs if the difference in initial levels of the interacting waves is significant. As the intensities of the oscillations at the different frequencies approach each other, the level of transfer into the high frequency range decreases and the direction of the process begins to depend on the relative phase orientation of the disturbances, as determined by the net initial phases difference $\Phi_0 = \psi_1 - \psi_2 - \psi_3$. Maximum transfer phases were found ($\Phi_0{}^m \sim 5\pi/4$ for symmetric and $\pi/2$ for asymmetric triplets) with minimum transfer occurring into the high frequency range at ($\Phi_0{}^m + \pi$). Between these values at steps of $\pi/2$ lie phases of prohibited nonlinear transfer, where amplitude development is defined by linear laws.

Despite the fact that the growth in amplitude of the various modes for the maximum transfer phases is of the same order of magnitude, the wave intensities (the quantities recorded in amplitude–frequency spectrograms) will be significantly different. We will demonstrate how mean square intensity is calculated, this being a quantity analogous to the degree of turbulence of the disturbed flow. By definition $I_j = [(< u'^2 > + < v'^2 > + < w'^2 >)/3U_0{}^2]^{1/2}$. For the waves of Eq. (1), normalized to a characteristic U_0 at the nozzle section $I_j = B_j e^{-\alpha_j{}^i x} T_j$, where the calculated value $T_j = [(|u_j|^2 + |v_j|^2 + |w_j|^2)/6]^{1/2}$ is obtained at $|p|_{max} = 1$ for all modes. Note also that for linear waves the amplitudes B_j are constant, while their linear dynamics are determined by an exponential factor. For nonlinear couplings A_j can be determined from system (2).

The distributions of maximum T values, characterizing the properties of both linear and slightly nonlinear waves are shown in Fig. 3 by the solid lines for $Sh_2 = 0.125$ and dashed lines for $Sh_1 = 0.25$. It is evident that for the axisymmetric modes the calculated T values decrease down the flow, which leads to a decrease in the linear intensities I. On the other hand, for the spiral modes the T values down the flow increase, especially at the higher frequency, which should lead to an increase in the linear intensities. Combining the dynamic characteristics of A and T for the triplets under consideration, we obtain a pattern of wave intensity distribution, to which we can now give a physical interpretation.

One calculation variant is shown in Fig. 4 for symmetric (a) and nonsymmetric (b) triplets. The initial wave intensities at the lower frequency were chosen identical here ($I_2{}^0 = I_3{}^0 = 5\%$), so that the net perturbation intensity at the first peak comprises 10%. Waves at the lower frequency for a given low level $I_1{}^0$ of the higher frequency wave evolve by a practically linear law, so that the level of the first peak remains unchanged. For the higher frequency wave the initial intensity level $I_1{}^0$ was chosen to be of background order [$I_2{}^0/200$ or $(I_2{}^0 + I_3{}^0)/400$], and maximum transfer phases were considered.

It is evident from Fig. 4 that despite the significant increase in amplitude in the symmetric triplet, that because of the reduction T_1, the intensity of the axisymmetric wave I_1 increases only by a factor of four times as compared to the initial level. on the other hand, for the nonsymmetric triplet the nonlinear interaction can increase the intensity of the spiral wave by two orders of magnitude, while the linear level (dashed lines) can increase by a factor of 20 times at the end of the interval studied.

This permits the conclusion that the mechanism of three-wave resonant intensification may be a realistic technique for separating the significant amplification of spiral modes from background oscillations over the initial segment of the supersonic jet.

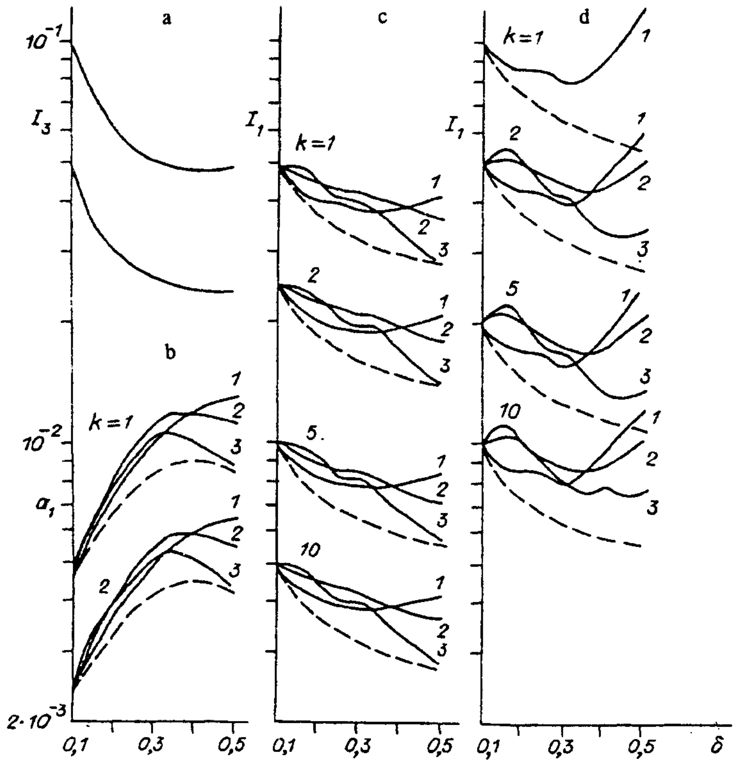

Fig. 5

One more problem was considered while studying three-wave interactions – that of generation of traveling longitudinal velocity waves. As was shown in [8, 9], within the initial segment of a nonisobaric jet there exists yet one more form of unstable oscillations – Taylor–Gortler (T–G) standing vortex waves, formed because of curvature of the gas motion trajectories within the centrifugal force field produced by the cellular structure of the initial segment. The spectral composition of such natural disturbances indicates a superpositional set of simple harmonic waves with various azimuthal numbers ($1 \leq n \leq 100$).

The change in spectrum down the flow permits the conclusion that the low-mode components of the T–G waves damp out quite rapidly. However it is just those waves with low azimuthal numbers which can enter into resonant interaction with the travelling coarse-scale shear waves, affecting their amplitude. In the present study we will consider resonant coupling in one of the possible triplets. For $M_0 = 1.5$ and $Sh_{1,2} = 0.25$ such a triad consists of the Mach mode $n_1 = 1$, $n_2 = -1$, and the T–G wave with $n_3 = 2$, for which $\alpha^r{}_3$ and $\omega_3 \sim 0$. The mathematical provisions for modeling are the same as for the triplets considered previously.

It was also found that interaction in this resonant triad is also possible for transfer of energy from T–G waves to traveling waves, as well as for the preferential net initial phases. We ill illustrate transfer for three favorable phases; $\Phi_0 = 5\pi/4$, $6\pi/4$, and $7\pi/4$ (lines 1-3 of Fig. 5). The initial intensity of the T–G wave was defined by values of 5 and 10%. The initial traveling wave intensity $I_1{}^0 = I_2{}^0$ corresponded to values of $I_3{}^0/k$, where k = 1, 2, 5, 10. It developed that for any initial traveling wave intensity in these phases the longitudinal dynamics of T–G waves is determined solely by linear rules (Fig. 5a), i.e., development of a steady wave is independent of the presence of traveling waves in the flow. Physically, this is easily explained by the fact that the value of the centrifugal forces creating T–G waves is independent of the degree of flow disturbance.

From Fig. 5b, it is evident that for $a_1 = |A_1|$ as compared to linear (dashed lines) for $I_1{}^0 = 5\%$ (k = 1) and $I_1{}^0 = 2.5\%$ (k = 2). The thickness $\delta = 2$ corresponds approximately to the end of the first cell of the non-isobaric jet. Because of the a_1 intersection intervals, it is difficult to assign a preference to one of the phases considered. For $\Phi_0 = 5\pi/4$ maximum transfer is achieved at the end of the cell, while at $\Phi_0 = 6\pi/4$ this occurs at the midpoint, and for $\Phi_0 = 7\pi/4$, at the nozzle section itself.

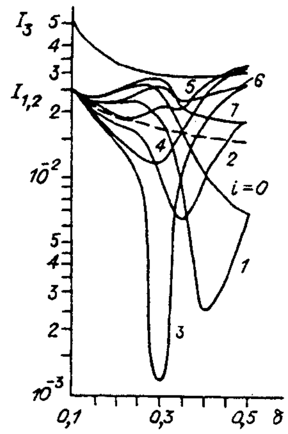

Fig. 6

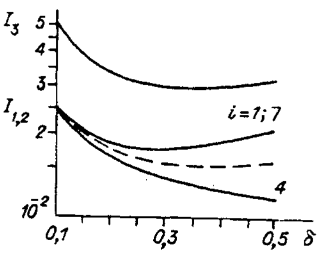

Fig. 7

From Figs. 5b,c, it is evident that for $I_3^0 = 5$ and 10% that intensification of the traveling wave is independent of the initial level, and is determined solely by the initial intensity of the T−G wave I_3^0, i.e., occurs within a parametric amplification regime. At the end point of the interval studied for $\Phi_0 = 5\pi/4$ the one wave is intensified 1.5 times, and the other, 2.15 times. The variant considered here corresponds to a curvature $I/R_0 = 0.05$.

Similar results were found for this same triplet at lower radii of curvature ($R_0 = 10$). Figure 6 shows longitudinal distributions of T−G wave intensity ($I_3^0 = 5\%$) and Mach wave intensity ($I_{1,2}^0 = I_3^0/2$) for various initial phases $\Phi_0 = (\pi/4)i$, defined by the net initial wave phase shift. With increase in curvature the degree of influence on the traveling wave increases somewhat, but its character remains unchanged.

The presence of regions of significant reduction in traveling wave intensity as compared to the linear levels permits the proposition that the literally reproduced longitudinal turbulence may suppress, rather than intensify shear instability travelling waves.

The effect for lower frequencies at $Sh_{1,2} = 0.125$ proved to be even less, which corresponds to the first peak in the band of defined frequencies described above Figure 7 shows that for this frequency the effect of the T−G wave on the Mach wave is limitingly small.

The modeling we have performed permits the conclusion that the experimentally observed reduction in intensity of the Taylor−Gortler mode n = 2 is controlled by intrinsic dynamics, upon which Mach mode shear waves have practically no effect. For those latter waves, the presence in the initial segment of a stationary longitudinal turbulence creates conditions for some growth, achieved in a parametric resonance regime given favorable phase relationships among the waves under consideration.

The relative phases found above for which significant reduction in amplitude and intensity of traveling waves occurs allow us to assume the presence of longitudinal vortices to be a stabilizing factor, which under certain conditions inhibits longitudinal growth of disturbances in the wide-band spectrum of a supersonic jet.

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